

**BCA / M-19**  
**MATHEMATICAL FOUNDATION-II**  
**Paper-BCA-123**

*Time allowed : 3 hours]*

*[Maximum marks : 80]*

*Note : Attempt five questions in all. Question No. 1 is compulsory. Attempt four more questions selecting exactly one question from each unit. All questions carry equal marks.*

**Compulsory Question**

1. Explain following:  $8 \times 2 = 16$
- (a) Truth table
  - (b) Mathematical induction
  - (c) Group
  - (d) Cosets
  - (e) Singular matrix
  - (f) Rank of a matrix
  - (g) Eigen vector
  - (h) Skew-Hermitian matrix.

**Unit-I**

2. (a) Show that  $\sim(p \Leftrightarrow q) \equiv p \Leftrightarrow \sim q$  8
- (b) Using the principle of mathematical induction, prove that for all  $n \in \mathbb{N}$ ,  $11^{n+2} + 12^{2n+1}$  is divisible by 133. 8

3. State and prove the laws of logic.

**Unit-II**

4. (a) If  $(G, \cdot)$  be a group; then solve the equation  $a \cdot x \cdot a = b$  in  $G$  8
- (b) Let  $H$  be a subgroup of group  $G$  and define  $N(H) = \{\alpha \in G : \alpha H = H\alpha\}$ . Prove that  $N(H)$  is a subgroup of  $G$  8
5. (a) Let  $H = \{5x : x \in Z\}$  be a subgroup of  $I$ . Prepare the composition table for  $Z/H$ . 8
- (b) Let  $D$  be an integral domain and  $F$  be a field. Define a mapping  $\psi : D \rightarrow F$  such that  $\psi(\alpha) = (\alpha, 1)$  for all  $\alpha \in D$ . Then show that  $\psi$  is an isomorphism of  $D$  into  $F$ . 8

**Unit-III**

6. Find  $A^{-1}$ , where  $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ . Hence, solve the system of linear equations:
- $$x + 2y - 3z = -4$$
- $$2x + 3y + 2z = 2$$
- $$3x - 3y - 4z = 11$$

(3)

7. Solve the following system of equations :

$$x - y + 2z - 3w = 0$$

$$3x + 2y - 4z + w = 0$$

$$4x - 2y + 9w = 0$$

16

#### Unit-IV

8. (a) Prove that the eigen values of a triangular matrix are the diagonal elements of the matrix. 8

- (b) Prove that any two characteristic vectors corresponding to two distinct characteristic roots of a Hermitian matrix are orthogonal. 8

9. Diagonalize, if possible, the matrix 16

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 7 & -4 \\ 9 & 1 & 3 \end{bmatrix}$$

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